C4 Paper A – Marking Guide

1.	$= \frac{2x}{2x^2+3x-5} \times \frac{x^2-x}{x^3}$		M1	
	$= \frac{2x}{(2x+5)(x-1)} \times \frac{x(x-1)}{x^3}$		M1	
	$= \frac{2}{x(2x+5)}$		M1 A1	**(4)**
2.	$4x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$		M1 A1	
	$\frac{dy}{dx} = 0 \quad \therefore\ 4x + y = 0, \quad y = -4x$		M1 A1	
	sub. $2x^2 - 4x^2 - 16x^2 + 18 = 0$		M1	
	$x^2 = 1,\ x = \pm 1 \ \therefore\ (-1, 4), (1, -4)$		A2	**(7)**
3.	*(i)*	$(1 + ax)^n = 1 + nax + \frac{n(n-1)}{2}(ax)^2 + \ldots$	B1	
		$\therefore\ an = -4, \qquad \frac{a^2n(n-1)}{2} = 24$	B1	
		$\Rightarrow\ a = \frac{-4}{n}$, sub. $\Rightarrow\ \frac{16}{n^2} \times \frac{n(n-1)}{2} = 24$	M1 A1	
		$8(n-1) = 24n, \qquad n = -\frac{1}{2},\ a = 8$	M1 A1	
	(ii)	$(1 + 8x)^{-\frac{1}{2}} = \ldots + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(8x)^3 + \ldots$	M1	
		$\therefore\ k = -\frac{5}{16} \times 512 = -160$	A1	**(8)**
4.	*(i)*	$= \left\lvert \frac{1\times 6+5\times 3+(-1)\times(-6)}{\sqrt{1+25+1}\times\sqrt{36+9+36}} \right\rvert$	M1 A1	
		$= \frac{27}{\sqrt{27}\times\sqrt{81}} = \frac{\sqrt{27}}{9} = \frac{3\sqrt{3}}{9} = \frac{1}{3}\sqrt{3}$	M1 A1	
	(ii)	$\sin(\angle AOB) = \sqrt{1-(\frac{1}{3}\sqrt{3})^2} = \sqrt{\frac{2}{3}}$	M1	
		area $= \frac{1}{2} \times 3\sqrt{3} \times 9 \times \sqrt{\frac{2}{3}} = \frac{27}{2}\sqrt{2}$	M1 A1	
	(iii)	$= OA \times \sin(\angle AOB) = 3\sqrt{3} \times \sqrt{\frac{2}{3}} = 3\sqrt{2}$	M1 A1	**(9)**
5.	*(i)*	$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$		
		$= \frac{\cos x\times\cos x - \sin x\times(-\sin x)}{\cos^2 x}$	M1 A1	
		$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$		
		$= \frac{1}{\cos^2 x}$	M1	
		$= \sec^2 x$	A1	
	(ii)	$\frac{dy}{dx} = 2 \times \tan x + 2x \times \sec^2 x = 2\tan x + 2x\sec^2 x$	M1 A1	
		$x = \frac{\pi}{4},\ y = \frac{\pi}{2}$, grad $= 2 + \pi$	B1	
		$\therefore\ y - \frac{\pi}{2} = (2 + \pi)(x - \frac{\pi}{4})$	M1	
		at P, $x = 0$		
		$\therefore\ y = \frac{\pi}{2} - \frac{\pi}{4}(2 + \pi) = -\frac{1}{4}\pi^2$	M1 A1	**(10)**

6.	*(i)*	$= \int (\text{cosec}^2 2x - 1)\ dx$	M1	
		$= -\frac{1}{2}\cot 2x - x + c$	M1 A1	
	(ii)	$u^2 = x + 1 \Rightarrow x = u^2 - 1,\ \frac{dx}{du} = 2u$	M1	
		$x = 0 \Rightarrow u = 1,\ x = 3 \Rightarrow u = 2$	B1	
		$I = \int_1^2 \frac{(u^2-1)^2}{u} \times 2u\ du = \int_1^2 (2u^4 - 4u^2 + 2)\ du$	M1 A1	
		$= [\frac{2}{5}u^5 - \frac{4}{3}u^3 + 2u]_1^2$	M1	
		$= (\frac{64}{5} - \frac{32}{3} + 4) - (\frac{2}{5} - \frac{4}{3} + 2) = 5\frac{1}{15}$	M1 A1	**(10)**
7.	*(i)*	$\int \frac{1}{(x-6)(x-3)}\ dx = \int 2\ dt$	M1	
		$\frac{1}{(x-6)(x-3)} \equiv \frac{A}{x-6} + \frac{B}{x-3},\quad 1 \equiv A(x-3) + B(x-6)$	M1	
		$x = 6 \Rightarrow A = \frac{1}{3},\ x = 3 \Rightarrow B = -\frac{1}{3}$	A2	
		$\frac{1}{3}\int (\frac{1}{x-6} - \frac{1}{x-3})\ dx = \int 2\ dt$		
		$\ln\lvert x-6\rvert - \ln\lvert x-3\rvert = 6t + c$	M1 A1	
		$t = 0,\ x = 0\ \therefore \ln 6 - \ln 3 = c,\quad c = \ln 2$	M1 A1	
		$x = 2 \Rightarrow \ln 4 - 0 = 6t + \ln 2$	M1	
		$t = \frac{1}{6}\ln 2 = 0.1155 \text{ hrs} = 0.1155 \times 60 \text{ mins} = 6.93 \text{ mins} \approx 7 \text{ mins}$	A1	
	(ii)	$\ln\left\lvert\frac{x-6}{2(x-3)}\right\rvert = 6t,\quad t = \frac{1}{6}\ln\left\lvert\frac{x-6}{2(x-3)}\right\rvert$		
		as $x \to 3,\ t \to \infty\ \therefore$ cannot make 3 g	B2	**(12)**
8.	*(i)*	$x = 1\ \therefore -1 + 4\cos\theta = 1,\ \cos\theta = \frac{1}{2},\ \theta = \frac{\pi}{3}, \frac{5\pi}{3}$	M1	
		$y > 0\ \therefore \sin\theta > 0\ \therefore \theta = \frac{\pi}{3}$	A1	
	(ii)	$\frac{dx}{d\theta} = -4\sin\theta,\quad \frac{dy}{d\theta} = 2\sqrt{2}\cos\theta$	M1	
		$\therefore \frac{dy}{dx} = \frac{2\sqrt{2}\cos\theta}{-4\sin\theta}$	M1 A1	
		at P, grad $= -\frac{2\sqrt{2}\times\frac{1}{2}}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{2}}{2\sqrt{3}}$	M1	
		grad of normal $= \frac{2\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{6}$	A1	
		$\therefore y - \sqrt{6} = \sqrt{6}(x - 1)$	M1	
		$y = \sqrt{6}x$, when $x = 0, y = 0\ \therefore$ passes through origin	A1	
	(iii)	$\cos\theta = \frac{x+1}{4},\ \sin\theta = \frac{y}{2\sqrt{2}}$	M1	
		$\therefore \frac{(x+1)^2}{16} + \frac{y^2}{8} = 1$	M1 A1	**(12)**
			Total	**(72)**